Analytical design formulas of TMD/TMDI for vortex-induced vibration control of bridges

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SUMMARY:
Tuned mass damper (TMD) and tuned mass damper inerter (TMDI) are promising devices for vortex-induced vibration (VIV) control of long-span bridges. However, existing analytical formulas of TMD/TMDI will lead to a suboptimal design result due to the neglection of nonlinear aeroelastic effect of VIV. This study proposes analytical formulas of TMD/TMDI that are suitable for VIV control of bridges. Governing equations of the bridge-TMD/TMDI system are established. The equivalent damping of TMD/TMDI to the bridge is derived. Then, the design formulas for TMD/TMDI frequency and damping ratio are developed. The reliability of the proposed formulas is validated by comparing with numerical results. The advantage of the proposed formulas is also validated through comparison with existing TMD/TMDI formulas. The proposed formulas can achieve a better control efficiency for the same predetermined structural parameters and have high accuracy within the scope of practical engineering applications.

Keywords: vortex-induced vibration; control; TMD; TMDI; design formulas

1. INTRODUCTION

VIV is a commonly occurred fluid-structural interaction phenomenon for flexible bridges (Xu et al., 2020). A cyclic large-amplitude or high-frequency oscillation threatens the serviceability and fatigue life of the bridge and may cause the failure of structural elements. Besides, VIV of the bridge is prone to occur at moderate or low wind velocities which can be easily achieved at the site of the bridge. Therefore, VIV suppression therefore has received intensive concerns in the bridge and wind engineering communities.
To suppress the VIV of bridges, TMD/TMDI have be validated to be one of the simplest and most effective devices for bridge VIV control, in which, TMDI can be treated as a generalization of TMD (it can degrade to TMD when the inertance of TMDI is zero). However, all the existing design formulas for TMD or TMDI are derived neglecting the influence of nonlinear aeroelastic effect during VIV, which may cause a suboptimal design result as thus a suboptimal control performance for VIV.
In this paper, analytical design formulas of TMD/TMDI for bridge VIV control are proposed. The reliability of the proposed formulas is validated by comparing with numerical results. The advantage of the proposed formulas is also validated through comparison with existing TMD/TMDI formulas. The proposed formulas can achieve a better control efficiency for the same predetermined structural parameters and have high accuracy within the scope of practical engineering applications.

2. BRIDGE-TMDI SYSTEM

A bridge equipped with a TMDI is shown in Fig. 1. In which, $y(x,t)$ is vertical displacement of the deck at position x; x_{tmdi} the location where the TMDI mass block is installed; $x_{inerter}$ the location where the other terminal of the inerter is linked to; m_t, k_t, c_t and b the mass, stiffness, damping, and inertance of TMDI, respectively; $y_t(t)$ the displacement of TMDI.

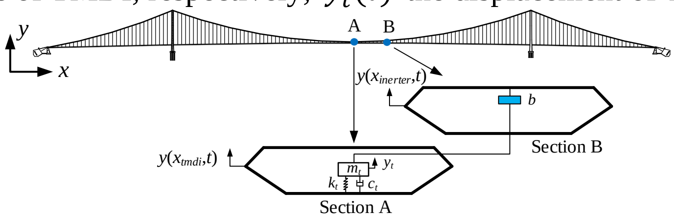

Figure 1. Bridge-TMDI system

2.1. Governing Equations

The equations of motion for the bridge and the TMDI can then be expressed as:

$$M\left[1+\phi_j(x_{tmdi})^2\mu+\phi_j(x_{tmdi})^2(1-\Delta\phi)^2\beta\right]\ddot{q}+2M\omega_b\xi_b\dot{q}+M\omega_b^2q$$
$$+M\left[\phi_j(x_{tmdi})\mu+\phi_j(x_{tmdi})(1-\Delta\phi)\beta\right]\Delta\ddot{q}_t=F(U,q,\dot{q},t) \quad (1.1)$$

$$\left[\phi_j(x_{tmdi})\mu+\phi_j(x_{tmdi})(1-\Delta\phi)\beta\right]\ddot{q}+(\mu+\beta)\Delta\ddot{q}_t+2(\mu+\beta)\omega_t\xi_t\Delta\dot{q}_t$$
$$+(\mu+\beta)\omega_t^2\Delta q_t=0 \quad (1.2)$$

where M is modal mass; L length of the girder; ω_b and ξ_b modal frequency and damping ratio of the bridge, respectively; $F(U,q,\dot{q},t)$ VIV force; $\Delta q_t(t)$ relative displacement between mass block and the deck; $\mu=m_t/M$ the mass ratio; $\beta=b/M$ the inertance ratio; ω_t the TMDI frequency; ξ_t the TMDI damping ratio; $\phi_j(x)$ and $q(t)$ modal shape and generalized coordinate of the *jth* mode, respectively; $\Delta\phi=\phi_j(x_{inerter})/\phi_j(x_{tmdi})$ the modal shape ratio.

2.2. Equivalent Damping Ratio of TMDI

Based on generalized governing equations of the bridge-TMDI system, Krylov-Bogoliubov method is adopted to calculate VIV response of the girder with and without TMDI, as shown in Eq. (2) and Eq. (3), respectively:

$$\int_0^{2\pi}F(U,q,\dot{q},t)sin\tau d\tau=-2\pi A\omega M\,\omega_b\xi_b \quad (2.1)$$

$$\int_0^{2\pi}F(U,q,\dot{q},t)cos\tau d\tau/(\pi AM)=\omega_b^2-\omega^2=0 \quad (2.2)$$

$$\int_0^{2\pi}F(U,q,\dot{q},t)sin\tau d\tau=-2\pi A\omega M\omega_b\left\{\xi_b+\frac{\gamma^4\phi_j^2(x_{tmdi})[\mu+(1-\Delta\phi)\beta]^2[(\mu+\beta)\xi_t\Omega]}{[(\mu+\beta)\Omega^2-(\mu+\beta)\gamma^2]^2+[2(\mu+\beta)\xi_t\gamma\Omega]^2}\right\} \quad (3.1)$$

$$\frac{\gamma^4\phi_j^2(x_{tmdi})[\mu+(1-\Delta\phi)\beta]^2[(\mu+\beta)\Omega^2-(\mu+\beta)\gamma^2]}{[(\mu+\beta)\Omega^2-(\mu+\beta)\gamma^2]^2+[2(\mu+\beta)\xi_t\gamma\Omega]^2}+(\gamma^2-1)$$
$$+\phi_j^2(x_{tmdi})[\mu+(1-\Delta\phi)^2\beta]\gamma^2=0 \quad (3.2)$$

where A and ψ are vibration amplitude and phase; ω vibration frequency of the control system;

$\Omega = \omega_t/\omega_b$; $\gamma = \omega/\omega_b$.

Comparing Eq. (2.1) with Eq. (3.1), it can be noted that the contribution of TMDI is equivalent to increasing the damping ratio of the girder:

$$\xi_{eq} = \frac{\gamma^4 \phi_j^2(x_{tmdi})[\mu+(1-\Delta\phi)\beta]^2[(\mu+\beta)\xi_t\Omega]}{[(\mu+\beta)\Omega^2-(\mu+\beta)\gamma^2]^2+[2(\mu+\beta)\xi_t\gamma\Omega]^2} \tag{4}$$

3. ANALYTICAL DESIGN FORMULAS

The equivalent damping ratio contributed by TMDI can be expressed as the increment of Scruton number, that is, $\tilde{S}_c = 2m\xi_{eq}/(\rho D^2)$. It is assumed that the frequency ratio γ is zero. Therefore, the analytical design formulas of TMDI can be derived as:

$$\Omega = \sqrt{\frac{c_1 c_3^2\left[\frac{(\tilde{S}_c\rho D^2)^2}{(m^2 c_1^2)}+1\right]-c_2 c_3}{c_1 c_3^2\left[\frac{(\tilde{S}_c\rho D^2)^2}{(m^2 c_1^2)}+1\right]}} \quad \text{and} \quad \xi_t = \frac{c_2 c_3 \tilde{S}_c \rho D^2}{2c_1 m\sqrt{c_1 c_3^2\left[\frac{(\tilde{S}_c\rho D^2)^2}{(m^2 c_1^2)}+1\right]-c_2 c_3}\cdot\sqrt{c_1 c_3^2\left[\frac{(\tilde{S}_c\rho D^2)^2}{(m^2 c_1^2)}+1\right]}} \tag{5}$$

where

$$c_1 = \phi_j^2(x_{tmdi})[\mu + (1-\Delta\phi)^2\beta];\quad c_2 = \phi_j^2(x_{tmdi})[\mu + (1-\Delta\phi)\beta]^2;\quad c_3 = (\mu+\beta) \tag{6}$$

4. VALIDATION AND COMPARISION OF ANALYTICAL FORMULAS

4.1. Validation Through Numerical Result

TMDI will degrade to TMD once the inertance becomes zero. Dai et al. (2019) optimized the TMD parameters (Ω and ξ_t) using a numerical optimization algorithm. The equivalent damping of TMD (ξ_{eq}) is calculated by Eq. (4). The calculated TMD parameters using the proposed method in this study are compared with the numerical results, as shown in Fig. 2. The calculated curves of frequency ratio, damping ratio and equivalent damping ratio for TMD using the analytical formulas are almost coincide with those obtained using numerical method, suggesting that the proposed formulas have enough accuracy for deriving these equations are reasonable.

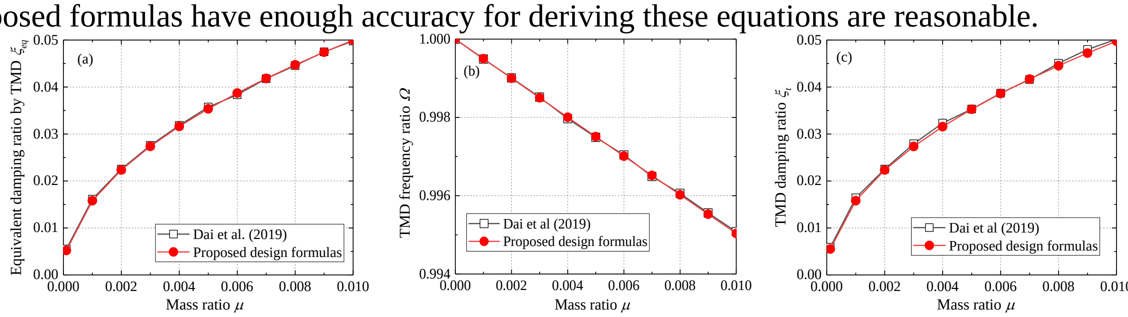

Figure 2. Comparison with numerical optimization method for TMD design. (a) equivalent damping ratio by TMD; (b) frequency ratio of TMD; (c) damping ratio of TMD.

4.2. Comparison with Other Design Formulas of TMDI

To further validate the capacity of the proposed formulas, the TMDI design results are compared with those designed by the formulas in Marian and Giaralis (2014) and Hu and Chen (2015). The above formulas are listed in Table 1.

Table 1. Design formulas of TMDI parameters

	TMDI frequency ratio Ω	TMDI damping ratio ξ_t
Marian-Giaralis	$\dfrac{\sqrt{[\beta(\mu-1)+(2-\mu)(1+\mu)]}}{(1+\beta+\mu)\sqrt{2(1+\mu)}}$	$\dfrac{\sqrt{\beta+\mu}\sqrt{\beta(3-\mu)+(4-\mu)(1+\mu)}}{2\sqrt{2(1+\beta+\mu)[\beta(1-\mu)+(2-\mu)(1+\mu)]}}$
Hu-Chen (H_∞)	$\dfrac{\sqrt{\mu+(1+\mu)\beta}}{(1+\mu)\sqrt{\mu+\beta}}$	$\sqrt{\dfrac{3\mu^2}{8(1+\mu)(\mu+\beta)}}$
Hu-Chen (H_2)	$\sqrt{\dfrac{2\beta(1+\mu)+\mu(\mu+2)}{2(1+\mu)^2(\mu+\beta)}}$	$\sqrt{\dfrac{(\mu+\beta)+\Omega^4(\mu+\beta)(1+\mu)^2-\Omega^2\mu(2+\mu)-2\beta\Omega^2(1+\mu)}{4\Omega^2(\mu+\beta)(1+\mu)}}$

Fig. 3 shows the equivalent damping ratio of TMDI. As can be seen, the equivalent damping ratio of TMDI increases with the increment of mass ratio and decreases with the increment of inertance ratio, indicating that the control efficiency of TMDI increases with the increment of mass ratio and decreases with the increment of inertance ratio. The formulas in this study can obtain largest values of equivalent damping ratio compared with others.

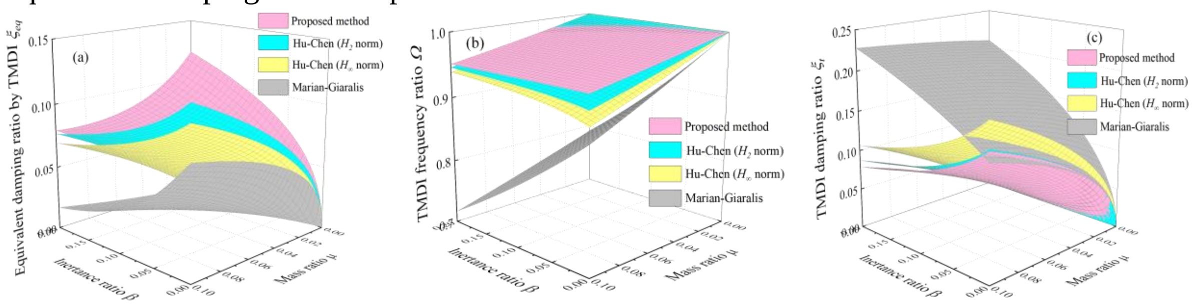

Figure 3. Comparison with other design formulas for TMDI design. (a) equivalent damping ratio by TMDI; (b) frequency ratio of TMDI; (c) damping ratio of TMDI.

5. CONCLUSIONS

Analytical design formulas of TMDI for suppressing VIV of bridges are proposed. The formulas can consider the influence of mechanical damping of the bridge and the installation arrangement of TMDI. The numerical optimization results are used to verify the accuracy of the design formula. Compared to the existing design formulas of TMDI, the proposed formulas can obtain a higher equivalent damping value for the same predetermined TMDI parameters.

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